from (11) we get —

$$n_{e1} = \frac{ik E_1 \sin^2\theta}{-4\pi e} + \frac{ik \cos^2\theta E_1}{-4\pi e}$$

$$n_{e1} = -\frac{ik E_1}{4\pi e}$$

substituting this value of n_{e1} in (10) we get —

$$-\frac{\omega k E_1}{4\pi e} + n_0 ik \left[\sin\theta\, v_{e1x} + \cos\theta\, v_{e1z}\right] = 0 \quad (12)$$

from (7), (8), (9) & (12) elliminating E_1, v_{e1x}, v_{e1y}, v_{e1z} we get —

$$\begin{vmatrix} \frac{e}{m_e}\sin\theta & -i\omega & \omega_e & 0 \\ 0 & +\omega_e & +i\omega & 0 \\ \frac{e}{m_e}\cos\theta & 0 & 0 & -i\omega \\ -\frac{\omega k}{4\pi e} & n_0 ik\sin\theta & 0 & n_0 ik\cos\theta \end{vmatrix} = 0$$

$$\Rightarrow (-1)^{3+4}(-i\omega)\begin{vmatrix} \frac{e}{m_e}\sin\theta & -i\omega & \omega_e \\ 0 & \omega_e & i\omega \\ -\frac{k\omega}{4\pi e} & ikn_0\sin\theta & 0 \end{vmatrix}$$

$$+ (-1)^{4+4}(ikn_0\cos\theta)\begin{vmatrix} \frac{e}{m_e}\sin\theta & -i\omega & \omega_e \\ 0 & \omega_e & i\omega \\ \frac{e}{m_e}\cos\theta & 0 & 0 \end{vmatrix} = 0$$

$$\Rightarrow i\omega\left\{\frac{e}{m_e}\sin\theta\left(k\omega n_0\sin\theta\right) + i\omega\left(\frac{i\omega^2 k}{4\pi e}\right) + \omega_e\left(-\frac{\omega^2 k e}{4\pi e}\right)\right\}$$

$$+ \cancel{ikn_0\cos\theta}\left\{\cancel{\frac{e}{m_e}\sin\theta \times 0}\right.$$

$$+ \cancel{\frac{e}{m_e}}\ ikn_0\cos\theta\left\{\frac{e}{m_e}\cos\theta\left(-\omega^2 e^2 + \omega^2\right)\right\} = 0$$

from (11) we get —

$$\begin{vmatrix} \frac{e}{m_e}\sin\theta & -i\omega & \omega_e & 0 \\ 0 & \omega_e & i\omega & 0 \\ \frac{e}{m_e}\cos\theta & 0 & 0 & -i\omega \\ -\frac{\omega k}{4\pi e} & n_0 i k\sin\theta & 0 & n_0 i k\cos\theta \end{vmatrix} = 0$$

$$\Rightarrow (-1)^{3+4}(-i\omega) \begin{vmatrix} \frac{e}{m_e}\sin\theta & -i\omega & \omega_e \\ 0 & \omega_e & i\omega \\ -\frac{\omega k}{4\pi e} & n_0 i k\sin\theta & 0 \end{vmatrix}$$

$$+ (-1)^{4+4}(n_0 i k\cos\theta) \begin{vmatrix} \frac{e}{m_e}\sin\theta & -i\omega & \omega_e \\ 0 & \omega_e & i\omega \\ \frac{e}{m_e}\cos\theta & 0 & 0 \end{vmatrix} = 0$$

$$\Rightarrow i\omega\left[\frac{e}{m_e}\sin\theta(+\omega n_0 k\sin\theta) + i\omega\left(\frac{i\omega^2 k}{4\pi e}\right) + \omega_e\left(\frac{\omega k\omega_e}{4\pi e}\right)\right]$$

$$+ n_0 i k\cos\theta\left[\frac{e}{m_e}\cos\theta\left(\omega^2 - \omega_e^2\right)\right] = 0$$

$$\Rightarrow \frac{i\omega^2 n_0 k e}{m_e}\sin^2\theta - i\frac{\omega^4 k}{4\pi e} + \frac{i\omega^2\omega_e^2 k}{4\pi e} +$$

$$\frac{i n_0 k e}{m_e}\omega^2\cos^2\theta - \frac{i n_0 k e}{m_e}\cos^2\theta\,\omega_e^2 = 0$$

$$\Rightarrow \frac{\omega^2 n_0 e}{m_e} - \frac{\omega^4}{4\pi e} + \frac{\omega^2\omega_e^2}{4\pi e} - \frac{n_0 e\,\omega_e^2\cos^2\theta}{m_e} = 0$$

$$\Rightarrow \omega^2\omega_e^2 - \omega^4 + \omega^2\frac{n_0 e^2 4\pi}{m_e} - \frac{n_0 e^2 4\pi}{m_e}\omega_e^2\cos^2\theta$$

$$\Rightarrow \omega^2\omega_e^2 - \omega^4 + \omega^2\omega_p^2 - \omega_p^2\omega_e^2\cos^2\theta$$

$k_x = k\sin\theta$, $k_y = k\cos\theta$

$$\left[\omega_p^2 = \frac{4\pi e^2 n_0}{m_e}\right]$$

$$\Rightarrow \omega^2(\omega_c^2 + \omega_p^2) - \omega^4 - \omega_p^2\omega_c^2\cos^2\theta = 0$$

$$\Rightarrow \omega^2\underline{\omega_h^2} - \omega^4 - \omega_p^2\omega_c^2\cos^2\theta = 0$$

$$\Rightarrow \omega^2(\omega^2 - \omega_h^2) + \omega_p^2\omega_c^2\cos^2\theta = 0$$

which is required dispersion relation —

$$\Rightarrow \omega^2\omega_c^2 + \omega^2\omega_p^2 - \omega^4 - \omega_p^2\omega_c^2\cos^2\theta = 0$$

$$\Rightarrow \omega^2(\omega_c^2 + \omega_p^2) - \omega^4 - \omega_p^2\omega_c^2\cos^2\theta = 0$$

$$\Rightarrow \omega^2 \underbrace{\omega_h^2}_{\text{upper hybrid frequency}} - \omega^4 - \omega_p^2\omega_c^2\cos^2\theta = 0$$

$$\Rightarrow \boxed{\omega^2(\omega^2 - \omega_h^2) + \omega_p^2\omega_c^2\cos^2\theta = 0} \quad (13)$$

which is the required dispersion relation

from (13) we get —

$$(\omega^2)^2 - \omega^2\omega_h^2 + \omega_p^2\omega_c^2\cos^2\theta = 0$$

$$\omega^2 = \frac{(\omega_h^2) \pm \sqrt{\omega_h^4 - 4\omega_p^2\omega_c^2\cos^2\theta}}{2}$$

where θ tends to zero we get $\rightarrow$

$$\omega^2 = \frac{\omega_h^2 \pm \sqrt{\omega_h^4 - 4\omega_p^2\omega_c^2}}{2}$$

$$= \frac{(\omega_c^2 + \omega_p^2) \pm \sqrt{\{(\omega_c^2 + \omega_p^2)^2 - 4\omega_p^2\omega_c^2\}}}{2}$$

$$= \frac{(\omega_c^2 + \omega_p^2) \pm (\omega_c^2 - \omega_p^2)}{2}$$

Taking +ve sign

$$\omega^2 = \omega_c^2$$

Taking −ve sign

$$\omega^2 = \omega_p^2$$

Taking θ tends to $\frac{\pi}{2}$

$$\omega^2 = \frac{\omega_c^2 + \omega_p^2 \pm (\omega_c^2 + \omega_p^2)}{2}$$

Taking +ve

$$\omega^2 = \omega_p^2 + \omega_c^2$$

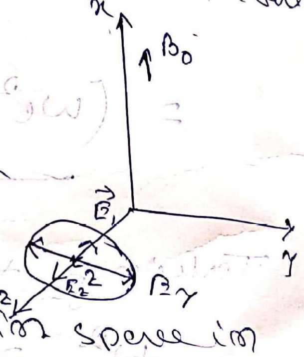
Taking -ve sign

$$\omega^2 = 0$$

(*) Dispertion relation of a electromagnetic wave propagating in magnetized electron plasma.

To derive the dispertion relation of an electromagnetic wave propagating in magnetized electron plasma. we assume that the following assumptions

i) There is static magnetic field B_0 in the direction ‖el to the x-axis.



ii) there are no thermal motion $T_e = 0$

iii) the ions are fixed in space in a uniform distribution.

iv) the plasma is cold an infinite in extent.

v) The wave propagates ~~into~~ a plasma in such a direction that it ~~[illegible]~~ develope of it component E_x along $\vec{k}$ does becoming longitudinal and possibly transverse

Thus we have —

$$\vec{E}_1 = E_y \hat{y} + E_z \hat{z}$$

and also for the magnetic field $\vec{B}_1 = B_y\hat{y} + B_z\hat{z}$

The plasma is neutral at rest

$$\vec{v}_0 = 0, \quad \vec{E}_0 = 0$$

From the above set of assumptions we have the following electron eqns of motion

$$\frac{\partial \vec{v}_e}{\partial t} + (\vec{v}_e \cdot \vec{\nabla})\vec{v}_e = -\frac{e}{m_e}\vec{E} - \frac{e}{m_e c}(\vec{v}_e \times \vec{B}) \quad \text{— (1)}$$

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \quad \text{— (2)}$$

$$\vec{\nabla} \times \vec{B} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c}\vec{J}$$

$$= \frac{1}{c}\frac{\partial \vec{E}}{\partial t} - \frac{4\pi e n_e \vec{v}_e}{c} \quad \text{— (3)}$$

Also we have the following linearized approximations [$\because$ electron current density $J = -e n_e v_e$]

$$\vec{v}_e = \vec{v}_0 + \vec{v}_{e1}$$

$$\vec{B} = \vec{B}_0 + \vec{B}_1$$

$$\vec{E} = \vec{E}_0 + \vec{E}_1$$

$$n_e = n_0 + n_1$$

therefore, the linearized form of (1) to (3) are —

$$\frac{\partial \vec{v}_{e1}}{\partial t} = -\frac{e\vec{E}_1}{m_e} - \frac{e}{m_e c}(\vec{v}_{e1} \times \vec{B}_0) \quad \text{— (4)}$$

$$\vec{\nabla} \times \vec{E}_1 = -\frac{1}{c}\frac{\partial \vec{B}_1}{\partial t} \quad \text{— (5)}$$

$$\vec{\nabla} \times \vec{B}_1 = \frac{1}{c}\frac{\partial \vec{E}_1}{\partial t} - \frac{4\pi e n_0 v_{e1}}{c} \quad \text{— (6)}$$

Now writing the eqns (4) to (6) components

$$\frac{\partial v_{e1y}}{\partial t} = -\frac{e}{m_e} E_{1y} - \frac{e B_0}{m_e c} v_{e1z} \quad \text{--- (7)}$$

$$\frac{\partial v_{e1z}}{\partial t} = -\frac{e}{m_e} E_{1z} + \frac{e}{m_e c} v_{e1y} B_0 \quad \text{--- (8)}$$

$$+\frac{\partial E_{1z}}{\partial x} = +\frac{1}{c}\frac{\partial B_{1y}}{\partial t} \quad \text{--- (9)}$$

$$\frac{\partial E_{1y}}{\partial x} = -\frac{1}{c}\frac{\partial B_{1z}}{\partial t} \quad \text{--- (10)}$$

$$\frac{\partial B_{1z}}{\partial x} = -\frac{1}{c}\frac{\partial E_{1y}}{\partial t} + \frac{4\pi e n_0 v_{e1y}}{c} \quad \text{--- (11)}$$

$$\frac{\partial B_{1y}}{\partial x} = \frac{1}{c}\frac{\partial E_{1z}}{\partial t} - \frac{4\pi e n_0 v_{e1z}}{c} \quad \text{--- (12)}$$

Now differentiating (10) w.r.t x and using (11) we get.

$$\frac{\partial^2 E_{1y}}{\partial x^2} = -\frac{1}{c}\frac{\partial}{\partial x}\left(\frac{\partial B_{1z}}{\partial t}\right)$$

$$= -\frac{1}{c}\frac{\partial}{\partial t}\left(\frac{\partial B_{1z}}{\partial x}\right)$$

$$= -\frac{1}{c}\frac{\partial}{\partial t}\left[-\frac{1}{c}\frac{\partial E_{1y}}{\partial t} + \frac{4\pi e n_0 v_{e1y}}{c}\right]$$

$$\Rightarrow \left(\frac{\partial^2}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) E_{1y} + \frac{4\pi e n_0}{c^2}\frac{\partial v_{e1y}}{\partial t} \quad \text{--- (13)}$$

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Differentiating (9) w.r.t x using (12) we get —

$$\frac{\partial^2 E_{1z}}{\partial x^2} = \frac{1}{c}\frac{\partial}{\partial t}\left(\frac{\partial B_{1y}}{\partial x}\right)$$

$$= \frac{1}{c}\frac{\partial}{\partial t}\left[\frac{1}{c}\frac{\partial E_{1z}}{\partial t} - \frac{4\pi e n_0 v_{1z}}{c}\right]$$

$$\Rightarrow \left[\frac{\partial^2}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right]E_{1z} + \frac{1}{c^2}4\pi e n_0 \frac{\partial v_{1z}}{\partial t} = 0 \quad \text{—— (14)}$$

from (7) we get —

$$\frac{\partial v_{e1y}}{\partial t} + \Omega_e v_{e1z} = -\frac{e}{m_e}E_{1y} \quad \text{—— (15)}$$

where

$$\Omega_e = \frac{eB_0}{m_e c}$$

electron plasma frequency

from (8) we have

$$\frac{\partial v_{e1z}}{\partial t} - \Omega_e v_{e1y} = -\frac{e}{m_e}E_{1z} \quad \text{—— (16)}$$

multiplying (15) by Ω_e and differentiating (16) w.r.t t & adding we get

$$\Omega_e\frac{\partial v_{e1y}}{\partial t} + \Omega_e^2 v_{e1z} + \frac{\partial^2 v_{e1z}}{\partial t^2} - \Omega_e\frac{\partial v_{e1y}}{\partial t}$$

$$= -\frac{e\Omega_e}{m_e}E_{1y} - \frac{e}{m_e}\frac{\partial E_{1z}}{\partial t}$$

$$\Rightarrow \left[\frac{\partial^2}{\partial t^2} + \Omega_e^2\right]v_{e1z} = -\frac{e}{m_e}\left[\Omega_e E_{1y} + \frac{\partial E_{1z}}{\partial t}\right] \quad \text{—— (17)}$$

differentiating (15) w.r.t t and multiplying (16) by Ω_e and substituting we get -